# OPTICAL ROTATORY POWER <br> OF THE FLEXIBLE AROMATIC COMPOUNDS <br> ( $S$ )-( - )-1-PHENYLETHANOL AND ( $(S)$-( - )-1-(2-METHYLPHENYL)ETHANOL* 

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#### Abstract

The method allowing to describe the Cotton effects of the compounds with perturbed symmetry of the aromatic chromophore caused by incorporation of the chiral substituent was applied to the model compounds ( $S$ )-( - )-1-phenylethanol and ( $S$ )-( - )-1-(2-methylphenyl)ethanol. The preferred conformations were located upon using the empirical potential functions. Then the optical rotatory power was computed as a sum of the contributions of one-electron and coupled oscillator mechanisms. It can be derived from the results obtained, that the cause of the opposite signs of the Cotton effect of the ${ }^{1} L_{\mathrm{b}}$ band at the same absolute configuration of the both compounds is not a change of the preferred conformation but a varying symmetry of the transition. This symmetry change is accompanied by a change of orientation of the transition moments in the both compounds studied.


Recently the development of the semiempirical quantum chemical methods has enabled a rather detailed understanding of the optical activity phenomena ${ }^{1}$. However, most of the papers reported deal with the compounds possessing carbonyl chromophore ${ }^{2-5}$ and aliphatic compounds respectively, in which the direction of the electric transition moments corresponds to the spatial orientation of $\sigma$ bonds ${ }^{6,7}$.

The calculation of the optical rotatory power of aromatic chiral compounds represents the more complicated case because of the necessity to approximate the wavefunctions of their excited states with the aid of several singly excited configurations and due to the lack of information concerning their magnetic transition moments. For these reasons most of the works on the aromatic chromophore were performed within the basis of Kuhn's ${ }^{8}$ and Kirkwood's ${ }^{9}$ theory of coupled oscillators for which the knowledge of electric transition moments of at least two isolated chromophores in the optically active molecule is sufficient ${ }^{10-15}$. In the case of a compound which does not belong to the class of rigid molecules a determination of preferred conformations should precede the calculation of the optical rotatory power. This

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type of calculation has been reported by Hooker and Schellman ${ }^{16}$ on interpreting ${ }^{1} L_{\mathrm{b}}$ and ${ }^{1} L_{\mathrm{a}}$ Cotton effects of $o$-, $m$ - and $p$-tyrosines.
In our laboratory we are studying the asymmetric reductions of alkyl aryl ketones ${ }^{17-19}$ leading to partially optically active alcohols of 1 -arylethanol type. The optical purity of the products obtained amounts up to 50 per cent and enambles CD measurements at least within the region of ${ }^{1} L_{\mathrm{b}}$ absorption band of the aromatic nucleus. In this paper we try to interpret the ${ }^{1} L_{\mathrm{b}}$ Cotton effects of $(S)-(-)$-1-phenylethanol $(I)$ and of $(S)-(-)-1-(2$-methylphenyl)ethanol (II). Both compounds, which are very similar in respect of structure, exhibit inverse Cotton effects at the same absolute configuration in the region under consideration. Thus, it remains to be explained whether this inversion consists in a change of conformational population or in a change of the orientation of transition moments caused by a perturbation of the chromophore with a methyl group.

## EXPERIMENTAL

Partially optically active ( $R$ )-(+)-1-phenylethanol $[\alpha]_{\mathbb{D}}^{20} 20.40^{\circ}$; optical purity $52.2 \%$ and ( $R$ )-$-(+)-1-\left(2\right.$-methylphenyl)ethanol $[\alpha]_{\mathrm{D}}^{20} 10 \cdot 30^{\circ}$; optical purity $14.4 \%$ were prepared by asymmetric reduction of acetophenone and methyl o-tolyl ketone respectively upon using the chiral agent prepared from equimolar amounts of lithium aluminium hydride and ( - )-quinine ${ }^{17.18}$. The absorption spectra and CD curves (recalculated for optically pure compounds) are given in Fig. 1. The $(R)-(+)$ absolute configuration was established by Mislow ${ }^{20}$ for 1-phenylethanol upon applying the chemical correlation to lactic acid and suggested for 1 -( 2 -methylphenyl)ethanol on the basis of qualitative stereochemical analysis of the course of asymmetric reduction ${ }^{18}$. The theoretical calculations were executed for $(S)-(-)$ enantiomers of both compounds and consequently the experimental values of rotatory strength were converted in this sense. The CD curve of ( $S$ )-$-(-)$-1-phenylethanol ( $I$ ) convering the bands ${ }^{1} L_{\mathrm{b}}$ and ${ }^{1} L_{\mathrm{a}}$ was reported previously by Verbit ${ }^{21}$ and for the band ${ }^{1} L_{\mathrm{b}}$ also by Gillard and Mitchell ${ }^{22}$. The lastly mentioned authors assigned, however, the $(S)-(+)$ configuration to 1-phenylethanol without any further discussion.

Fig. 1
CD (—) and UV ( $-\cdots$ ) Spectra of $(R)-(+)$-1-Phenylethanol $(I)$ and of $(R)-(+)-$ -1-(2-Methylphenyl)ethanol (II)


UV spectra were recorded on CF4 Optica Milano and CD curves on Roussell Jouan Dichrographe.

## Calculation of Preferred Conformations

The preferred conformations of the compounds $I$ and $I I$ have not been determined experimentally up to the present. Only Quellette, Marks and Miller ${ }^{23}$ effectuated a study of conformations of alkyl aryl methanols by means of NMR spectroscopy of corresponding dimethyl sulphoxidic solutions. Three possible preferred conformations $I a, I b, I c$ of the compound $I$ were considered within this study and the conformation Ia exhibiting benzylic hydrogen coplanar to aromatic ring was chosen as a starting point. It is probable, that this conformation is energetically the most advantageous one following the analysis of the molecular models.


I


II


Ia


Ib


Ic

Moreover, the authors demonstrated the possible existence of a weak hydrogen bond connecting the $\pi$-electron system and the hydroxyl group.

The molecular conformation of $I$ may be described in terms of three independent torsional angles and in the case of compound $I I$ even of four ones. For that reason we upstained from a detailed calculation of conformational hypersurfaces with the aid of quantum chemical methods taking into account all valence electrons. These calculations were performed by means of simple empirical potential functions. We applied the method described by Hooker and Schellmann ${ }^{16}$ used by them for the interpretation of the preferred conformations of tyrosines. We calculated the contributions of nonbonded pairwise interactions and that of electrostatic interactions using Lennard-Jones 6-12 potential (1) and a function of coulombic type (2) respectively; $r_{i j}$ represents the interatomic distance, $K=332$ the conversional factor
and $\varepsilon$ the apparent dielectric constant of solvent. For this constant we have used the value of unity in a same way as did Hooker and Schellmann ${ }^{16}$. The values given by Ooi and Scheraga ${ }^{24}$ were used for the parameters $a_{\mathrm{ij}}, b_{\mathrm{ij}}$ and the atomic charges $q$ were obtained within the summation of the contributions $\pi$ and $\delta$ calculated by the method HMO and Del Re respectively. In the calculation according to HMO we con-

$$
\begin{gather*}
V_{\mathrm{i} j}^{\mathrm{nb}}=-a_{\mathrm{i} j} r_{\mathrm{ij}}^{-6}+b_{\mathrm{ij}} r_{\mathrm{ij}}^{-12}  \tag{1}\\
V_{\mathrm{ij}}^{\mathrm{elst}}=K q_{\mathrm{i}} q_{j} \varepsilon^{-1} r_{\mathrm{ij}}^{-1} \tag{2}
\end{gather*}
$$

sidered but the inductive influence of aliphatic groups on the coulomb integral of the corresponding carbon atom ( $\delta \alpha$ for methyl group $-0.5 ; \delta \alpha$ for 1 -hydroxyethyl group -0.4 ). A standard set of parameters ${ }^{25}$ was used for computations of $\sigma$-electron distribution by the method of Del Re. We neglected the term describing the barriers to internal rotation (the exchange interaction of electrons localized in orbitals on the atoms connected by a bond), as this term cannot influence qualitatively the results. The computations performed upon using the relations (1) and (2) do not involve the energy of solvation. In addition, the both contributions mentioned above cannot be summed up due to the uncertainty in the chosen value of the apparent dielectric constant ${ }^{16}$.

The starting atomic coordinates were calculated from standard bond lengths and bond angles tabulated by Pople and Beveridge ${ }^{26}$. The initial geometry $\varphi_{1}=0^{\circ}$ and $\varphi_{2}=0^{\circ}$ of $(S)-(-)$-1-phenylethanol $(I)$ is given in Fig. 2. Numerical calculations were tabulated within the range of $0-180^{\circ}$ (angle $\varphi_{1}$ ) and $0-360^{\circ}$ (angle $\varphi_{2}$ ), the step applied being $15^{\circ}$. The map of nonbonded interactions thus obtained is given in Fig. 3 A . The value of $\varphi_{2}$ except of regions of $\varphi_{1}$ about $0^{\circ}$ and $180^{\circ}$ showing a considerable interaction of hydrogen atoms in the ortho positions of the benzene ring with those of the hydroxyl group does not exhibit serious influence on the total


Fig. 2
Starting Geometry of (S)-(-)-1-Phenylethanol (I), $\varphi_{1}=0^{\circ} ; \varphi_{2}=0^{\circ}$
nonbonded energy. Quite the same conclusions can be derived for $\varphi_{2}=0^{\circ}, 180^{\circ}$ based on two sections of the conformational map Fig. 3B. The shape of electrostatic interactions Fig. $3 C$ does not show any important geometrical dependence and consequently, it is neglected for the case of $I$. The minimum nonbonded interactions in $(S)$-( - -1-phenylethanol (I) occur at $\varphi_{1} \sim 135^{\circ}$, this position roughly corresponding to the preferred conformation $I a$ with the benzylic hydrogen coplanar to the aromatic nucleus. The computed rotational barrier is about $3 \mathrm{kcal} \mathrm{mol}^{-1}$.
$(S)-(-)$-1-(2-methylphenyl)ethanol (II), its initial conformation at zero values of the dihedral angles $\varphi_{1}$ and $\varphi_{2}$ being given in Fig. 4 exhibits four degrees of freedom with respect to the possible rotation of the methyl group in the ortho position of the aromatic ring. For that reason it seems necessary to organize the calculation in the way to obtain - at a reasonable consumption of the computing time - maximum information of the preferred conformations of this compound. Within the solution of this problem the calculations were executed in the range of $0-360^{\circ}$ for $\varphi_{1}$ at the



Fig. 4
Starting Geometry of (S)-(-)-1-(2-Methylphenyl)ethanol ( $I$ ) , $\varphi_{1}=0^{\circ} ; \varphi_{2}=0^{3}$
values of $0^{\circ}, 180^{\circ}$ for $\varphi_{2}$. The process was repeated three times for three different $a-c$ orientations of the methyl group in the ortho position. The results obtained are given in Fig. 5-7.


Considering the relations mentioned above and supposing energetically more advantageous $b, c$ orientations of the methyl group, $(S)-(-)-1-(2$-methylphenyl)ethanol (II) exhibits two energetically comparable minima at $\varphi_{1}=240^{\circ}(I I a)$ and $\varphi_{1}=$ $=315^{\circ}(I I b)$ respectively on the curves of nonbonded interactions. The conformations $I I a$ is, of course, less advantageous due to the proximity of the maximum on the curves of electrostatic interactions. The minimum found within the range of $\varphi_{1} \sim 315^{\circ}$ (IIb) corresponds to the preferred conformation $I a$ of $(S)-(-)-1$-phenylethanol $(I)$ conformably with compared Dreiding models of compounds $I$ and $I I$. The final decision concerning the relative preference of the conformations IIa and IIb of (S)-(-)-1-(2-methylphenyl)ethanol (II) would call for much more detailed calculations using probably more sophisticated method compared with the empirical potential functions here applied. Simple calculations given in this paper favour the conformation $I I b$ and indicate the preferred conformations of the both compounds studied to be approximately equal at least at the level of approximation used in this work.


## Calculation of Optical Rotatory Strength

The quantity, representing the extent to which the electronic transition from the state $A$ to $B$ is optically active, is the so-called rotatory strength defined according to Condon ${ }^{27}$ and Rosenfeld ${ }^{28}$ by the relation (3)

$$
\begin{equation*}
R_{A \rightarrow B}=\operatorname{Im}\{\langle A| \boldsymbol{\mu}|B\rangle .\langle B| \mathbf{m}|A\rangle\}, \tag{3}
\end{equation*}
$$

where $\boldsymbol{\mu}$ and $\mathbf{m}$ are the operators of electric and magnetic dipole moment respectively and

$$
\begin{equation*}
\operatorname{Im}\{a+i b\}=b \tag{4}
\end{equation*}
$$

Expressing the operator of electric dipole moment in the dipole velocity representation ${ }^{29}$ and limiting the calculation to the here important case, in which the wavefunction of the excited state $B$ is composed from two singly excited configurations $B_{1}$ and $B_{2}$ mixed together to the extent given by LCI coefficients $c_{\mathrm{B}_{1}}$ and $c_{\mathrm{B}_{2}}$ respectively, it is possible to write:

$$
\begin{equation*}
R_{A \rightarrow B}=-\frac{e^{2} \hbar^{2}}{4 \pi m^{2} c^{2}} \sum_{\mathrm{i}, \mathrm{j}}^{2} c_{\mathrm{B}_{\mathrm{i}}} c_{\mathrm{B}_{\mathrm{J}}} \frac{\langle A| \boldsymbol{\nabla}\left|B_{\mathrm{i}}\right\rangle}{\bar{v}_{A \rightarrow B_{\mathrm{i}}}}\left\langle B_{\mathrm{j}}\right| \mathbf{r} \times \mathbf{\nabla}|A\rangle . \tag{5}
\end{equation*}
$$

In this equation $\bar{v}_{A \rightarrow B}$ represents the wavenumber of the transition $A \rightarrow B$ in $^{-1}$, $\mathbf{r}$ is the position vector operator respective for the electron engaged in the transition and the symbols $e, \hbar, m, c$ hold their usual meaning.

According to the relation (3) the optical rotatory strength is given by the dot product of the matrix elements of electric and magnetic dipole operators. It is nonvanishing only in the case of nonzero values of the both matrix elements mentioned above. Only chiral molecules, which do not possess the rotation-reflection axis of symmetry, fulfill this condition. Hence, the wavefunction used in calculating the transition moments must also be a chiral one. Performing the actual calculation it is possible either to start from the wavefunction of the whole chiral molecule (EHT, CNDO) and to compute the optical rotatory strength in the direct process or to utilize the simplified symmetrical wavefunction of the chromophore. In the


Fig. 5
Nonbonded ( $A$ ) and Electrostatic ( $B$ ) Interactions in ( $S$ )-( - )-1-(2-Methylphenyl)ethanol (II) at $\left.\varphi_{2}=0^{\circ}(-)^{-}\right)$and $180^{\circ}(\cdots)$

The methyl group connected to the aromatic nucleus is in a conformation.


Fig. 6
Nonbonded ( $A$ ) and Electrostatic ( $B$ ) Interactions in (S)-(-)-1-(2-Methylphenyl)ethanol (II) at $\varphi_{2}=0^{\circ}$ ( - ) and $180^{\circ}(\cdots)$

The methyl group connected to the aromatic nucleus is in $b$ conformation.
latter case the origin of optical activity is simulated either with the use of the perturbational potential or in coupling the original wavefunction with another chromophore or polarizable group. Direct calculations based on CNDO (ref.4,5) and EHT (ref. ${ }^{3}$ ) wavefunctions were successfully applied to the compounds possessing carbonyl chromophore. In the region of aromatic chromophores there is reported only a paper of Bush ${ }^{30}$ related to the optical rotatory strengths of the both anomers of deoxyadenosine. This paper is based on EHT wavefunctions treated by the method of Gould and Hoffmann ${ }^{3}$. However this work does not take into account the configurational interaction and consequently the results reported are probably inaccurate. The optical rotatory strength of 1-methylindane was studied upon using the perturbational potential ${ }^{31}$ (one-electron mechanism ${ }^{32}$ ).

The one-electron mechanism, the $\mu-m$ mechanism and the coupling with polarizable groups according to Kirkwood ${ }^{9}$ may be considered in the case of compounds of 1 -arylethanol type. The lastly mentioned mechanism was successfully applied to CD data of nucleosides by Eyring and coworkers ${ }^{33-35}$. The $\mu-m$ mechanism is not involved in our calculations due to the lack of information related to the magnetic transition moments arising from the aliphatic part of the molecule.

We have calculated the optical rotatory strengths of ${ }^{1} L_{\mathrm{b}}$ transitions of $(S)-(-)$ --1-phenylethanol (I) and of $(S)$-( - )-1-(2-methylphenyl)ethanol (II). Within the computation, the optical rotatory strength was divided into contributions presented by the one-electron mechanism ${ }^{32}$ on the one side and by the Kirkwood mechanism of coupled oscillators ${ }^{9}$ on the other.


Fig. 7
Nonbonded (A) and Electrostatic (B) Interactions in (S)-(-)-1-(2-Methylphenyl)ethanol (II) at $\varphi_{2}=0^{\circ}(-)$ and $180^{\circ}(-\cdots)$

The methyl group connected to the aromatic nucleus is in $c$ conformation.


Fig. 8
Orientation of Toluenic (a) and $o$-xylenic (b) Chromophore Into the Coordinate System

The necessary information relating to the symmetry and configuration interaction of ${ }^{1} L_{\mathrm{b}}$ transition of the compounds $I$ and $I I$ was obtained in using PPP $\pi$-electron only wavefunctions of toluene and $o$-xylene respectively, supposing the direction of the electric transition moment to be determined predominantly by the - nearest surrounding of the chromophore. Further, the $C_{2 v}$ local symmetry of the both chromophores was considered. The results of the PPP calculations are given in Table I together with the computational details. The coordinate system used is given in Fig. 8.
It is clear, that the wavefunction of the first excited state is suitable for approximating by two configurations of the same symmetry participating in this transition nearly to the same extent. This result seems to be rather typical in the case of aromatic chromophores. The introduction of the methyl group into the ortho position results in a change of symmetry of the configurations involved and leads finally to a different orientation of the transition moments in the both compounds.

The computation rotatory strength with the use of the one-electron mechanism consists in replacing the wavefunctions of the chiral molecule by the perturbed chromophoric wavefunction obtained upon the action of the electrostatic perturbational potential. The transition moments of this wavefunction fulfill the condition of equation (3) and give rise to a nonzero value of rotatory strength. The nonvanishing components of the both moments can be found upon using standard group--theoretical methods. The calculation was started from the CNDO/2 wavefunctions of toluene and $o$-xylene as model compounds. The $C_{2 v}$ local symmetry of both chromophores was considered again. This intentional inaccuracy was used to suppress the influence of hydrogen atoms situated in the methyl groups of model compounds.

## Table I

PPP Calculation of Toluene and $o$-Xylene
Methyl groups were approximated with the aid of the hyperconjugative model and the following parametrization: $\mathrm{C}\left(s p^{2}\right): I=11.16 \mathrm{eV}, A=0.03 \mathrm{eV} ; \mathrm{C}\left(\mathrm{H}_{3}\right): I=11.42 \mathrm{eV}, A=0.58 \mathrm{eV}$; $\mathrm{H}_{3}(\mathrm{C}): I=9.50 \mathrm{eV}, A=0.00 \mathrm{eV} ; k_{\mathrm{C}-\mathrm{CH}_{3}}=0.51 ; k_{\mathrm{C}-\mathrm{H}_{3}}=1.25 ; \beta=-2.388 \mathrm{eV}$. On the aromatic nucleus the values of bond lengths and bond angles were taken as $1.4 \AA$ and $120^{\circ}$ respectively. Bond length $\mathrm{C}-\mathrm{CH}_{3}=1.52 \AA ; \mathrm{C}-\mathrm{H}=1.1 \AA$.

| Compound | $\lambda_{\max }{ }^{1} L_{\mathrm{b}}, \mathrm{nm}$ | $f$ | CI composition | Symmetry |
| :---: | :---: | :---: | :---: | :---: |
| Toluene | 256 | 0.0003 | $\begin{array}{lll}4-6 & 0.6993\end{array}$ | $B_{2}$ |
|  |  |  | $3-50.6793$ |  |
| $o$-Xylene | 260 | 0.0007 | $5-7 \quad 0.6910$ | $A_{1}$ |
|  |  |  | $4-6 \quad 0 \cdot 6606$ |  |

A survey of molecular orbitals and of singly excited configurations of toluene and $o$-xylene used in the calculation is given in Fig. 9 and 10.

The calculations were executed for the conformations $I a$ and $I I b$ of $(S)-(-)$ --1-phenylethanol (I) and of (S)-(-)-1-(2-methylphenyl)ethanol (II) respectively, supposing the oxygen atom to be the sole origin of the perturbation.

In the case of compound $I$, the wavefunction of the first excited state is approximated by two $B_{2}$ configurations contributing to the $x$-component of the electric transition moment and to the $z$-component of the magnetic one. Due to the probable affection of ${ }^{1} L_{\mathrm{b}}$ transition by energetically higher positioned $\pi-\pi^{*}$ transitions, the interactions with the $A_{1}$ states contributing to the electric transition moment in the $y$-direction were also considered. Moreover the $\pi-\sigma^{*}$ states $B_{1}\left(\mu_{\mathrm{z}}, m_{\mathrm{x}}\right)$ and $A_{2}\left(m_{\mathrm{y}}\right)$ were taken into account. The basis determined in this way is able to supply nonzero contributions to the optical rotatory strength along all the three coordinate axes. The terms used in the calculation of the transition moments of both conformations $B_{2}^{\mathrm{a}}$ and $B_{2}^{\mathrm{b}}$ were constituted with the use of the first order perturbation theory:

$$
\begin{align*}
&\langle G| \nabla\left|B_{2}^{\mathrm{a}}\right\rangle=\left(G|\nabla| B_{2}^{\mathrm{a}}\right)+\sum_{C=A_{1^{\mathrm{a}}, A_{1}{ }^{\mathrm{b}}, B_{1}{ }^{\mathrm{a}}, B_{2} \mathrm{~b}}} \frac{\left(C|\nabla| B_{2}^{\mathrm{a}}\right)}{\Delta E}(G|\nabla| C)=\sqrt{ } 2\left(\chi_{18}|\nabla| \chi_{20}\right)+ \\
&+2 \cdot \sqrt{ } 2\left[\frac{\left(\chi_{19}|V| \chi_{20}\right)}{\Delta E}\left(\chi_{18}|\nabla| \chi_{19}\right)+\frac{\left(\chi_{15}|V| \chi_{18}\right)}{\Delta E}\left(\chi_{15}|\nabla| \chi_{20}\right)+\right. \\
&+\frac{\left(\chi_{21}|V| \chi_{20}\right)}{\Delta E}\left(\mathrm{x}_{18}|\nabla| \chi_{21}\right)+\frac{\left(\chi_{22}|V| \mathrm{x}_{20}\right)}{\Delta E}\left(\chi_{18}|\nabla| \chi_{22}\right) \tag{6}
\end{align*}
$$



Fig. 9
CNDO/ 2 Orbitals of Toluene Used in the Calculation of Optical Rotatory Strength


Fig. 10
CNDO/2 Orbitals of $o$-xylene Used in the Calculation of Optical Rotatory Strength

$$
\begin{gather*}
\left\langle B_{2}^{\mathrm{a}}\right| \mathbf{r} \times \nabla|G\rangle=\left(B_{2}^{\mathrm{a}}|\mathbf{r} \times \nabla| G\right)+\sum_{C=B_{1}^{\mathrm{a}}, B_{1} \mathrm{~b}} \frac{\left(C|V| B_{2}^{\mathrm{a}}\right)}{\Delta E}(C|\mathbf{r} \times \nabla| G)  \tag{7}\\
\langle G| \nabla\left|B_{2}^{\mathrm{b}}\right\rangle=\left(G|\nabla| B_{2}^{\mathrm{b}}\right)+\sum_{C=A_{1^{\mathrm{a}}, A_{1} \mathrm{~b}}} \frac{\left(C|V| B_{2}^{\mathrm{b}}\right)}{\Delta E}(G|\nabla| C)  \tag{8}\\
\left\langle B_{2}^{\mathrm{b}}\right| \mathbf{r} \times \nabla|G\rangle=\left(B_{2}^{\mathrm{b}}|\mathbf{r} \times \nabla| G\right)+\sum_{C=A_{2^{\mathrm{a}}, A_{2} \mathrm{~b}}} \frac{\left(C|V| B_{2}^{\mathrm{b}}\right)}{\Delta E}(C|\mathbf{r} \times \nabla| G) \tag{9}
\end{gather*}
$$

The value of $\Delta E$ in the relations (6)-(9) is defined as:

$$
\begin{equation*}
\frac{(A|V| B)}{\Delta E}=\frac{(A|V| B)}{E_{\mathrm{B}}-E_{\mathrm{A}}} \tag{10}
\end{equation*}
$$

Round bracketts relate to unperturbed states. The molecular orbitals $\chi_{i}$ can be identified with the aid of Fig. 9. Similar terms were obtained for the compound II, in which the nonvanishing component of the electric dipole moment in the $y$-direction is supplied by the unperturbed $A_{1}$ states. The interaction with the $\pi$ - $\pi^{*}$ states $B_{2}$ gives rise to $\mu_{x}, m_{z}$ and the other components are supplied by the $\pi-\sigma^{*}$ states $B_{1}\left(\mu_{z}, m_{\mathrm{x}}\right)$ and $A_{2}\left(m_{y}\right)$. Hence, the transition moments of the $A_{1}^{\text {a }}$ configuration in the compound $I I$ are the following ones:

$$
\begin{align*}
& \langle G| \nabla\left|A_{1}^{\mathrm{a}}\right\rangle=\left(G|\nabla| A_{1}^{\mathrm{a}}\right)+\sum_{C=B_{2}{ }^{\mathrm{a}}, \boldsymbol{B}_{2^{\mathrm{b}}, B_{1}}} \frac{\left(C|V| A_{1}^{\mathrm{a}}\right)}{\Delta E}(G|\nabla| C),  \tag{11}\\
& \left\langle A_{1}^{\mathrm{a}}\right| \mathbf{r} \times \nabla|G\rangle=\sum_{C=B_{2}{ }^{\mathrm{a}}, B_{2}{ }^{\mathrm{b}}, B_{1}^{\mathrm{a}}, A_{2}^{\mathrm{a}}} \frac{\left(C|V| A_{1}^{\mathrm{a}}\right)}{\Delta E}(C|\mathbf{r} \times \nabla| G) . \tag{12}
\end{align*}
$$

Similar relations are incorporated in the calculation of the transition moments of the $A_{1}^{\mathrm{b}}$ configuration.

Numerical calculations of the matrix elements were executed on using atomic functions of carbon in the mono- $\xi$ form ${ }^{36}$ and a value of $\xi_{c} 1 \cdot 5679$. The sole nonzero integral of the $\boldsymbol{\nabla}$ operator within one-center ones is represented by $\langle 2 p z| \nabla|2 s\rangle$. Its value is easily obtainable by analytical computation ${ }^{5}$ and amounts to 0.4526 atomic units. Within two-center integrals it was necessary to compute the values of $\langle 2 p y| \nabla|2 p z\rangle,\langle 2 p z| \nabla|2 p z\rangle,\langle 2 p x| \mathbf{r} \times \nabla|2 p z\rangle,\langle 2 p y| \mathbf{r} \times \nabla|2 p z\rangle$. The values of the gradient integrals were determined on using the relations reported in the paper ${ }^{37}$. The integrals of the $\mathbf{r} \times \nabla$ operator were transformed to overlap integrals. Numerical values of these integrals are given in Table II. The integrals of the magnetic transition moments were expanded according to ${ }^{5}$ :

$$
\begin{equation*}
\langle A| \mathbf{r} \times \boldsymbol{\nabla}|B\rangle=\mathbf{R} \times\langle A| \boldsymbol{\nabla}|B\rangle+\langle A| \mathbf{r}_{\mathrm{A}} \times \boldsymbol{\nabla}|B\rangle . \tag{13}
\end{equation*}
$$

The particular integrals were first computed in the local coordinates and then transformed into the molecular coordinate system.
The electrostatic perturbational potential can be described by equation

$$
\begin{equation*}
V=\sum_{\mathbf{k}} q \|\left|\mathbf{r}-\mathbf{R}_{\mathrm{k}}\right| \tag{14}
\end{equation*}
$$

in which $q$ represents the charge localized in the centre of the perturbation, $\boldsymbol{r}$ is the position vector of electron related to this centre and $\mathbf{R}_{k}$ is the position vector of the atom influenced by the perturbation. The equation (14) can be expanded ${ }^{38}$ according

Table II
Two-center Integrals of the $\boldsymbol{\nabla}$ and $\mathbf{r} \times \nabla$ Operator Represented in Atomic Units

| Interatomic distance <br> $\AA$ | $\langle 2 p z\| \boldsymbol{\nabla}\|2 p z\rangle$ <br> $\langle 2 p y\| \boldsymbol{\nabla}\|2 p z\rangle$ | $\langle 2 p x\| \mathbf{r} \times \boldsymbol{\nabla}\|2 p z\rangle$ <br> $-\langle 2 p y\| \mathbf{~} \times \boldsymbol{\nabla}\|2 p z\rangle$ |
| :---: | :---: | :---: |
|  |  |  |
| 1.40 | 0.2263 | 0.2694 |
| 2.40 | 0.0448 | 0.0421 |
| 2.78 | 0.0218 | 0.0195 |

## Table III

Integrals of the $\langle A| V_{\mathrm{i}}|B\rangle$ Type
The values of $V_{\mathrm{i}} / q$ in atomic units are given in the table. The value of $q$ used in the calculation was -0.23 a.u. (the result of the calculation according to Del Re ). Carbon atoms of the aromatic nucleus are numbered according to Fig. 11.

| $i$ | $\left\langle 2 p z_{\mathrm{i}}\right\| V_{\mathrm{i}}\left\|2 p z_{\mathrm{i}}\right\rangle$ | $\left\langle 2 p z_{\mathrm{i}}\right\| V_{\mathrm{i}}\left\|2 p x_{\mathrm{i}}\right\rangle$ | $\left\langle 2 p z_{\mathrm{i}}\right\| V_{\mathrm{i}}\left\|2 p y_{\mathrm{i}}\right\rangle$ | $\left\langle 2 p z_{\mathrm{i}}\right\| V_{\mathrm{i}}\left\|2 s_{\mathrm{i}}\right\rangle$ |
| :---: | :---: | :---: | :---: | :---: |
|  | 0.2166 | 0.0074 | 0.0147 |  |
| 2 | 0.1460 | 0.0056 | 0.0068 | -0.0186 |
| 3 | 0.1111 | 0.0026 | 0.0047 | -0.0056 |
| 4 | 0.1056 | 0.0010 | 0.0048 | -0.0025 |
| 5 | 0.1246 | -0.0003 | 0.0066 | -0.0021 |
| 6 | 0.1819 | -0.0009 | 0.0127 | -0.0034 |
| 7 |  |  |  |  |

to the relation (15):

$$
\begin{align*}
& \left|\mathbf{r}-\mathbf{R}_{\mathrm{k}}\right|^{-1}=\sum_{\mathrm{t}=0}^{\infty} A_{\mathrm{t}}\left(\mathbf{r}, \mathbf{R}_{\mathrm{k}}\right) P_{\mathrm{t}}\left(\cos \gamma_{\mathrm{k}}\right)  \tag{15}\\
& A_{\mathrm{t}}\left(\mathbf{r}, \mathbf{R}_{\mathrm{k}}\right)=\left\{\begin{array}{lll}
r^{\mathrm{t}} / R_{\mathrm{k}}^{\mathrm{t}+1} & \text { for } & r<R_{\mathrm{k}} \\
R_{\mathrm{k}}^{\mathrm{t}} / r^{\mathrm{t}+1} & \text { for } & r>R_{\mathrm{k}}
\end{array}\right.
\end{align*}
$$

$P_{\mathbf{t}}\left(\cos \gamma_{k}\right)$ is the Legendre polynomial of the argument

$$
\begin{equation*}
P_{\mathrm{t}}\left(\cos \gamma_{\mathrm{k}}\right)=\cos \Theta \cos \Theta_{k}+\sin \Theta \sin \Theta_{k} \cos \left(\varphi-\varphi_{k}\right) \tag{16}
\end{equation*}
$$

The angles $\Theta_{k}$ and $\varphi_{k}$ are given in Fig. 11. The values of the matrix elements given in Table III were computed on using the relation

$$
\begin{align*}
& \left\langle\chi_{\mathrm{k}}\left(n l_{\mathrm{a}} m_{\mathrm{a}}\right)\right| V_{\mathrm{k}}\left|\chi_{\mathrm{k}}\left(n l_{\mathrm{b}} m_{\mathrm{b}}\right)\right\rangle= \\
& =q \sum_{\mathrm{t}=0}^{\infty} U_{\mathrm{l}}\left(\mathbf{R}_{\mathrm{k}}\right)(-1)^{\mathrm{m}_{\mathrm{a}}} \frac{2 \sqrt{ } \pi}{2 t+1} \sum_{l=\left|=\left|l_{\mathrm{a}}-l_{\mathrm{b}}\right|\right.}^{l+l_{\mathrm{b}} \mid}\left[\frac{\left(2 l_{\mathrm{a}}+1\right)\left(2 l_{\mathrm{b}}+1\right)}{2 l+1}\right]^{1 / 2} \times \\
& \times\left(l_{\mathrm{a}} l_{\mathrm{b}}-m_{\mathrm{a}} m_{\mathrm{b}} \mid l_{\mathrm{a}} l_{\mathrm{b}} l m\right)\left(l_{\mathrm{a}} l_{\mathrm{b}} 00 \mid l_{\mathrm{a}} l_{\mathrm{b}} l m\right) Y_{1}^{\mathrm{m}}\left(\Theta_{\mathrm{k}}, \varphi_{\mathrm{k}}\right) \delta_{1 \mathrm{t}} \tag{17}
\end{align*}
$$

In this relation $\left(l_{\mathrm{a}} l_{\mathrm{b}} m_{\mathrm{a}} m_{\mathrm{b}} \mid l_{\mathrm{a}} l_{\mathrm{b}} l m\right)$ are the Clebsch-Gordan coefficients,

$$
\begin{equation*}
U_{\mathrm{t}}=\int_{0}^{\infty} A_{\mathrm{t}}\left(\mathbf{r}, \mathbf{R}_{\mathrm{k}}\right) R^{2}(n, l) r^{2} \mathrm{~d} r \tag{18}
\end{equation*}
$$

and $\delta_{11}$ is the Kronecker symbol. The method reported above corresponds approximately with the partial point charge model elaborated by Stigter and Schellman ${ }^{39}$.

The transition moments of the perturbed states thus obtained were used in the calculation of the optical rotatory strength according to equation (5). Starting from


A


Fig. 11
Definition of the Angles
$A \varphi_{\mathrm{i}} ; B \theta_{\mathrm{i}}$ necessary for the calculation of the perturbational potential; (in Fig. 11A the oxygen atom is projected into the plane of the aromatic ring).
the electric transition moments the corresponding oscillator strengths were also computed. The resulting values of one-electron rotatory strengths given in Table IV agree in sign with the values obtained experimentally. It may be deduced from the relative magnitudes of the particular contributions to the value of the rotatory strength resulting from the combination of the transition moment components along the directions of the coordinate axes given in Table IV as $R_{x}, R_{y}, R_{z}$, that the largest contributions to the one-electron rotatory strength are exhibited by the moments acting in the directions $y$ and $x$ for the compounds $I$ and $I I$ respectively. These contributions arise from the $A_{1}$ (compound $I$ ) and $A_{2}$ (compound $I I$ ) states. Owing to the possible simulation of higher $\pi-\pi^{*}$ transitions in both compounds with the aid of these states, it is possible to interprete this fact as an influence of higher lying $\pi-\pi^{*}$ transitions on the optical rotatory strength of the ${ }^{1} L_{\mathrm{b}}$ band.
The perturbed transition moments of both compounds were utilized also in the calculation executed with the aid of coupled oscillators. The optical rotatory strength was calculated on using the modification of the Kirkwood method described by Eyring and coworkers ${ }^{34}$. The calculation is based on relations

$$
\begin{gather*}
R_{\mathrm{a}}=\sum_{\mathrm{b} \neq \mathrm{a}} \frac{2 \pi}{h c} \frac{v_{\mathrm{a}} v_{\mathrm{b}} \mu_{\mathrm{a}}^{2} \mu_{\mathrm{b}}^{2}}{\left(v_{\mathrm{b}}^{2}-v_{\mathrm{a}}^{2}\right)} G F  \tag{19}\\
G F=\left[\mathbf{e}_{\mathrm{a}} \cdot \mathbf{e}_{\mathrm{b}}-3 \frac{\left(\mathbf{e}_{\mathrm{a}} \cdot \boldsymbol{R}_{\mathrm{ab}}\left(\mathbf{e}_{\mathrm{b}} \cdot \boldsymbol{R}_{\mathrm{ab}}\right)\right)}{R_{\mathrm{ab}}^{2}}\right] \frac{\mathbf{e}_{\mathrm{a}} \cdot \mathbf{e}_{\mathrm{b}} \cdot \boldsymbol{R}_{\mathrm{ab}}}{R_{\mathrm{ab}}^{3}}, \tag{20}
\end{gather*}
$$

Table IV
Oscillator and Optical Rotatory Strength of ${ }^{1} L_{\mathrm{b}}$ Transitions of (S)-(-)-1-Phenylethanol (I) and (S)-(-)-1-(2-Methylphenyl)ethanol (II)

The values are given in c.g.s. units.

| Compound - conformation | Ia |  | IIb |  |
| :---: | :---: | :---: | :---: | :---: |
| $f_{\text {exp }}$. | $4 \cdot 70$ | $10^{-3}$ | $3 \cdot 19$ | $10^{-3}$ |
| $f_{\text {calc }}$. | $2 \cdot 46$ | $10^{-3}$ | $3 \cdot 60$ | $10^{-3}$ |
| $R$ - one-electron $\quad R_{\text {K }}$ | 4.70 | $10^{-44}$ | -4.52 | $10^{-43}$ |
| $R_{\text {y }}$ | $2 \cdot 14$ | $10^{-41}$ | $2 \cdot 18$ | $10^{-43}$ |
| $R_{\text {z }}$ | 7.35 | $10^{-43}$ | $-1.13$ | $10^{-42}$ |
|  | $2 \cdot 22$ | $10^{-41}$ | $-5.43$ | $10^{-42}$ |
| $R$ - coupled oscillators | $7 \cdot 13$ | $10^{-42}$ | -2.52 | $10^{-41}$ |
| $R_{\text {calc }}$. | $9 \cdot 35$ | $10^{-41}$ | $-2.57$ | $10^{-41}$ |
| $R_{\text {exp }}$. | 3.45 | $10^{-41}$ | $-1.08$ | $10^{-41}$ |

where $\mathbf{e}_{\mathrm{a}}$ is the unit vector in the direction of the electric transition moment, $\mu_{\mathrm{a}}$ its magnitude and $v_{\mathrm{a}}$ the transition frequency; the values of $\boldsymbol{e}_{\mathrm{b}}, \mu_{\mathrm{b}}, v_{\mathrm{b}}$ are related to the neighbouring polarizable group. $R_{\mathrm{ab}}$ represents the distance measured from the chromophore to the group $b$. The coupling of the transition moments of chromophore and of the moments localized in the bonds $\mathrm{C}-\mathrm{OH}$ and $\mathrm{C}-\mathrm{CH}_{3}$ was considered. This calculation was executed for the conformations $I a$ and $I I b$ too. The values tabulated in the work ${ }^{34}$ were assigned to the empirical parameters $\mu_{\mathrm{b}}, v_{\mathrm{b}}$. The results thus obtained are given in Table IV.

The total value of optical rotatory strength obtained upon adding the contributions of one-electron and dipole coupling mechanisms (Table IV) exhibits a correct sign for both compounds under study and its order of magnitude agrees with the value obtained experimentally. It can be concluded from the results obtained, that the reversed signs of the Cotton effects of ${ }^{1} L_{\mathrm{b}}$ band at equal absolute configuration are not caused by a change of the preferred conformation. The reason for this inversion is the change of the symmetry of the transition combined with the different orientation of the transition moments in both compounds. The remarkable influence of higher $\pi-\pi^{*}$ transitions on the rotatory strength of ${ }^{1} L_{\mathrm{b}}$ band is very interesting.

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